Statistical Approaches to Gas Distribution Modelling with Mobile Robots –
The Kernel DM+V Algorithm and Beyond

Many others … and Achim J. Lilienthal

AASS Learning Systems Lab, Örebro University
1. Introduction
2. Gas Distribution in a Natural Environment
3. Statistical Gas Distribution Modelling (GDM)
4. Kernel DM+V Algorithm
5. Importance of Pred. Variance for GDM
6. Kernel DM+V Extensions
7. Ongoing and Future Work
8. Summary
Gas Distribution Modelling with Stationary and Mobile Sensor Networks
1 GDM with Sensor Networks

Air Pollution Monitoring with Sensor Networks

The London Air Quality Network
(King’s College London)
Gas Distribution Modelling with Sensor Networks

- continuous collection of dense measurements
  - gas concentration, air flow, temperature, humidity
- central integration into one consistent, truthful model
- close integration with subsequent decision processes
1 GDM with Sensor Networks

- continuous collection of dense concentration measurements
- central integration into one consistent, truthful model
- close integration with subsequent decision processes
- integration of mobile sensors
  - positioned by human operators (field inspector) ...
GDM with Sensor Networks

- GDM with Autonomous Sensor Networks
  - continuous collection of dense concentration measurements
  - central integration into one consistent model
  - close integration with subsequent decision processes
  - integration of mobile sensors
    - positioned by human operators...
    - ... or by mobile robots
Advantages of Using Mobile Sensor Nodes

- fewer sensors are necessary (expensive ones can be used)
- compensation for inactive sensors
- optimisation of trajectory wrt the task (sensor planning)
  - adaptation to changing environmental conditions
  - possibility of source tracking
- integration into existing applications
- rapid deployment (less expensive)
- deployment at dangerous sites
- accurate positioning
Gas Distribution Modelling Projects at the AASS Learning System Lab – DustBot
1 GDM in AASS LSLab Projects

DustBot

- Networked and Cooperating Robots for Urban Hygiene
- Duration: December 1, 2006 – November 30, 2009 (36m)
- Coordinator: Scuola Superiore Sant'Anna (PSV, Pontedera)
- Partners: Italy, UK, Spain, Switzerland, Sweden
  - 5 universities/research institutes
  - 4 companies
- People involved at AASS
  - Matteo Reggente, Marco Trincavelli, Amy Loutfi, Achim Lilienthal, Todor Stoyanov (navigation)
1 GDM in AASS LSLab Projects

- DustBot
  - Networked and Cooperating Robots for Urban Hygiene
1 GDM in AASS LSLab Projects

- DustCart
1 GDM in AASS LS Lab Projects

- DustCart Scenario
1 GDM in AASS LSLab Projects

- DustBot
  - Networked and Cooperating Robots for Urban Hygiene
1 GDM in AASS LSLab Projects

- Pollution Monitoring in DustBot
  - continuous collection of concentration measurements
    - while performing other tasks
1 GDM in AASS LSLab Projects

- Pollution Monitoring in DustBot
  - Environmental Sensors

  Preconditioned sensors for
  - CO (0-100ppm)
  - NO₂ (0-200ppb)
  - O₃ (0-500ppb)
  - Accuracy 10%
  - ~ 200-300 €

  Temperature and Humidity sensor
  - ~ 20 €

Solid State Sensors
- Accuracy ~20%
- < 100 €

PM2.5-10 analyzer
- Accuracy 10%
- ~ 3000-3500 €
1 GDM in AASS LSLab Projects

- Pollution Monitoring in DustBot
  - experiments in realistic scenarios to come ...

- DustBot Demonstrations
  - Pontedera, Italy (May 9, 2009)
  - Peccioli, Italy (May/Jun)
  - Bilbao, Spain (Jun, 2009)
  - Örebro, Sweden (Jul 25, 2009)
  - Tomorrow City, Incheon (Aug, 2009)
    - with ETRI and KIST
Gas Distribution Modelling Projects at the AASS Learning System Lab – Diadem
1 GDM in AASS LSLab Projects

Diadem

- Distributed Information Acquisition and Decision-Making for Environmental Management
- Duration: September 1, 2008 – August 30, 2011 (36m)
- Coordinator: DECIS lab, Thales, The Netherlands
- Partners: Netherlands, Romania, Germany, Denmark, Belgium, Sweden (4 universities/research institutes, 5 companies)
- People involved at AASS
  - Sahar Asadi, Matteo Reggente, PhD, postdoc, Achim Lilienthal
1 GDM in AASS LSLab Projects

Diadem Prime Objective

- system to help making decisions to prevent chemical incidents or to mitigate their consequences by
  - establishing methods and tools that allow gaining a better understanding/overview of the distribution of gas that emanates as a result of a chemical incident,
  - providing easy access to available information (pre-processing sensor data and combining information from different sources)
  - connecting the people required to resolve a given situation and routing the appropriate information to them,
  - estimating consequences of alternative decisions and presenting them in form of a risk analysis to the decision-makers, and
  - automatic reasoning about the situation and suggesting actions
1 GDM in AASS LSLab Projects

- Diadem Scenario
  - Rotterdam harbour area
Gas Dispersal in a Natural Environment
2 Gas Dispersal in Natural Environments

- Chaotic Gas Dispersal
  - diffusion
  - advective transport
  - turbulent transport

[Smyth and Moum, 2001]
2 Gas Dispersal in Natural Environments

- Chaotic Gas Dispersal
2 Gas Dispersal in Natural Environments

- Turbulent Flow Characteristics
  - Turbulent transport is much faster than molecular diffusion
    - Gaseous ethanol at 25°C and 1 atm:
      - Diffusion constant: $0.119 \text{ cm}^2/\text{s}$ → Diffusion velocity: $20.7 \text{ cm/h}$
  - Turbulent flow is chaotic/unpredictable
    - Instantaneous velocity/concentration at some instant of time is generally insufficient to predict the velocity some time later
  - High degree of vortical motion
    - Large-scale eddies cause a meandering dispersal
    - Small scale eddies stretch and twist the gas distribution resulting in a complicated patchy structure

Statistical Gas Distribution Modelling
3 Statistical Gas Distribution Modelling

- Simulation of Turbulent Gas Distribution?

3 Statistical Gas Distribution Modelling

- Simulation of Turbulent Gas Distribution?
3 Statistical Gas Distribution Modelling

Simulation of Turbulent Gas Distribution?
3 Statistical Gas Distribution Modelling

- Simulation of Turbulent Gas Distribution?
  - computational fluid Dynamics (CFD) models?
Simulation of Turbulent Gas Distribution?

- no general solution to the fluid dynamics equations
- numerical simulations computationally expensive and depend sensitively on the initial/boundary conditions
- initial/boundary conditions not known in typical scenarios

→ model gas distribution statistically from a large number of measurements
3 Statistical Gas Distribution Modelling

- Statistical Gas Distribution Modelling
  - interpret concentration measurements statistically
    - statistical representation
      - gas sensor measurements treated as random variables
    - build a representation of the observed gas distribution from a sequence of measurements
3 Statistical Gas Distribution Modelling

- Statistical Gas Distribution Modelling
  - interpret concentration measurements statistically
    - statistical representation
      - gas sensor measurements treated as random variables
    - build a representation of the observed gas distribution from a sequence of measurements

- Problem Definition: Stat. Gas Distribution Modelling
  - learn predictive model
    \[
    p(r_* | \bar{x}_*, \bar{x}_{1:n}, r_{1:n})
    \]
Statistical Gas Distribution Modelling with Predictive Variance – Kernel DM+V Algorithm
Ext. Kernel Extrapolation DM (Kernel DM+V)

- GDM approached as a density estimation problem
  ⇒ relation to the Parzen window approach
Ext. Kernel Extrapolation DM (Kernel DM+V)

- GDM approached as a density estimation problem
- Gas sensor readings interpreted as a measure of the number of samples drawn from a particular grid cell
  \[ \Rightarrow \text{weigh measurement values with a kernel function} \]
- Measuring gas concentrations with a mobile robot
  \[ \neq \text{uniform sampling from a probability distribution} \]
  \[ \Rightarrow \text{normalisation to the density of measurements} \]
Ext. Kernel Extrapolation DM (Kernel DM+V)
- extension of Kernel DM
4 Kernel DM+V

Ext. Kernel Extrapolation DM (Kernel DM+V)

- extension of Kernel DM
- estimation of predictive mean and variance
- variance estimate depends on
  - true variance of measurements
  - distance to measurement locations

Estimating the Predictive Variance Entails a Significant Step Forward for Statistical GDM!

- the models better fit the structure of gas distributions
- allows model evaluation in terms of the data likelihood
  - learning meta parameters
  - comparison of different approaches to statistical GDM
- provides the means for
  - sensor planning
    (suggest new measurement locations based on the current model)
  - lazy update mechanisms
    (determine when the model should be updated or re-initialised)
4 Kernel DM+V

Experiments in the "Microscope Room"
Kernel DM+V

- Integrated weights

\[
\Omega_k = \sum_{i=1}^{\left| D \right|} \omega_{k,i}
\]

\[
\omega_{k,i} = \text{Gauss}\left( |\vec{x}_i - \vec{x}_k(c)|, \sigma \right)
\]

\[\sigma = 10 \text{ cm}\]
4 Kernel DM+V

Kernel DM+V

integrated weights

\[ \Omega_k = \sum_{i=1}^{|D|} \omega_{k,i} \]

\[ \omega_{k,i} = Gauss\left( |\vec{x}_i - \vec{x}_k(c)|, \sigma \right) \]

\( \sigma = 75 \text{ cm} \)
Kernel DM+V

- integrated weights, integrated readings

\[ \Omega_k = \sum_{i=1}^{\left| D \right|} \omega_{k,i} \]

\[ R_k = \sum_{i=1}^{\left| D \right|} \omega_{k,i} \cdot r_i \]
4 Kernel DM+V

Kernel DM+V

- integrated weights, integrated readings

\[ \Omega_k = \sum_{i=1}^{D} \omega_{k,i} \]
\[ R_k = \sum_{i=1}^{D} \omega_{k,i} \cdot r_i \]

- confidence map

\[ \alpha_k = 1 - \exp\left[-\frac{\Omega_k^2}{\sigma^2}\right] \]
4 Kernel DM+V

Kernel DM+V

- integrated weights, integrated readings
  \[ \Omega_k = \sum_{i=1}^{|D|} \omega_{k,i} \quad R_k = \sum_{i=1}^{|D|} \omega_{k,i} \cdot r_i \]

- confidence map
  \[ \alpha_k = 1 - \exp\left(-\frac{\Omega_k^2}{\sigma^2}\right) \]
  - for high values of \( \Omega_k \): \( \alpha_k \to 1 \)
  - for high values of \( \Omega_k = 0 \): \( \alpha_k = 0 \)
  - "high" and "low" relative to \( \sigma \)
4 Kernel DM+V

- Kernel DM+V
  - integrated weights

\[
\Omega_k = \sum_{i=1}^{\lvert D \rvert} \omega_{k,i}
\]

\[\sigma = 45 \text{ cm}\]
4 Kernel DM+V

- Kernel DM+V
  - confidence map

\[ \alpha_k = 1 - \exp \left[ -\frac{\Omega_k^2}{\sigma^2} \right] \]

\[ \sigma = 45 \text{ cm} \]
4 Kernel DM+V

Kernel DM+V

- integrated weights

\[ \Omega_k = \sum_{i=1}^{\\left|\rho\right|} \omega_{k,i} \]

\[ \sigma = 10 \text{cm} \]
Kernel DM+V

confidence map

\[ \alpha_k = 1 - \exp\left[-\frac{\Omega_k^2}{\sigma_\Omega^2}\right] \]

\( \sigma = 10 \text{ cm} \)
4 Kernel DM+V

Kernel DM+V

- integrated weights, integrated readings

\[ \Omega_k = \sum_{i=1}^{D} \omega_{k,i}, \quad R_k = \sum_{i=1}^{D} \omega_{k,i} \cdot r_i \]

- confidence map

\[ \alpha_k = 1 - \exp\left[ -\frac{\Omega_k^2}{\sigma^2} \right] \]

- predictive mean

\[ r_k = \alpha_k \cdot R_k / \Omega_k + \{1 - \alpha_k\} \cdot r_0 \]
\[ r_0 = \frac{1}{|D|} \sum_{i=1}^{D} r_i \]
Kernel DM+V – Example

predictive mean

\[ r_k = \alpha_k \cdot \frac{R_k}{\Omega_k} + \left\{1 - \alpha_k\right\} \cdot r_0 \]

\[ \sigma = 45 \text{ cm} \]
Kernel DM+V

- integrated weights, integrated readings
  \[ \Omega_k = \sum_{i=1}^{\lvert D \rvert} \omega_{k,i} \quad R_k = \sum_{i=1}^{\lvert D \rvert} \omega_{k,i} \cdot r_i \]
- confidence map
  \[ \alpha_k = 1 - \exp\left[-\frac{\Omega_k^2}{\sigma_\Omega^2}\right] \]
- predictive variance estimated separately
4 Kernel DM+V

- Kernel DM+V
  - integrated weights, integrated readings
    \[ \Omega_k = \sum_{i=1}^{\vert D \vert} \omega_{k,i} \quad R_k = \sum_{i=1}^{\vert D \vert} \omega_{k,i} \cdot r_i \]
  - confidence map
    \[ \alpha_k = 1 - \exp\left[-\frac{\Omega_k^2}{\sigma_\Omega^2}\right] \]
  - predictive variance estimated separately
  - variance contributions for each measurement
    \[ \tau_i = \left(r_i - r_{k(i)}\right)^2 \]
    \[ k(i) = \text{cell closest to } x_i \]
4 Kernel DM+V

Kernel DM+V

- integrated weights, integrated readings

\[ \Omega_k = \sum_{i=1}^{|D|} \omega_{k,i} \quad R_k = \sum_{i=1}^{|D|} \omega_{k,i} \cdot r_i \]

- confidence map

\[ \alpha_k = 1 - \exp\left[-\frac{\Omega_k^2}{\sigma^2}\right] \]

- predictive uncertainty

\[ v_k = \alpha_k \cdot V_k / \Omega_k + \{1 - \alpha_k\} \cdot v_0 \]

\[ \tau_i = \left(r_i - r_{k(i)}\right)^2 \quad V_k = \sum_{i=1}^{|D|} \omega_{k,i} \cdot \tau_i \quad v_o = \frac{1}{|D|} \sum_{i=1}^{|D|} \tau_i \]
Kernel DM+V – Example

predictive variance

\[ v_k = \alpha_k \cdot \frac{V_k}{\Omega_k} + \{1 - \alpha_k\} \cdot v_0 \]

\( \sigma = 45 \text{ cm} \)
4 Kernel DM+V – Remarks

Kernel DM+V

comparison with interpolation map

\[ \sigma = 45 \text{ cm} \]
4 Kernel DM+V – Remarks

- Comparison with Map from Trilinear Interpolation
  - MATLAB function `trisurf`
Kernel DM+V – Summary

\[ \omega_{k,i} = Gauss(\| \tilde{x}_i - \tilde{x}_k \|, \sigma) \]

\[ \Omega_k = \sum_{i=1}^{|D|} \omega_{k,i} \]

\[ \alpha_k = 1 - \exp \left[ -\Omega_k^2 / \sigma^2 \right] \]

\[ r_k = \alpha_k \cdot R_k / \Omega_k + \{1 - \alpha_k\} \cdot r_0 \]

\[ R_k = \sum_{i=1}^{|D|} \omega_{k,i} \cdot r_i \quad r_0 = \frac{1}{|D|} \sum_{i=1}^{|D|} r_i \]

\[ v_k = \alpha_k \cdot V_k / \Omega_k + \{1 - \alpha_k\} \cdot v_0 \]

\[ \tau_i = (r_i - r_{k(i)})^2 \quad V_k = \sum_{i=1}^{|D|} \omega_{k,i} \cdot \tau_i \quad v_o = \frac{1}{|D|} \sum_{i=1}^{|D|} \tau_i \]
4 Kernel DM+V – Remarks

- Complexity
  - $O[\text{training points} \times (\sigma/c)^2 + (L/c)^2 + \text{test points}]$

- Efficiency
  - Matlab implementation
### Parameter Selection
- 3 hyper-parameters: $c, \sigma, \sigma_\Omega$

### Observations
- Weak dependence on $c$
- $\sigma_\Omega$ can be related to $\sigma$

### Learning Parameters
- Cross-validation to find optimal hyper-parameters
- Fitness: negative log predictive density (NLPD)

\[
NLPD = \frac{1}{2|D|} \sum_{i=1}^{|D|} \left\{ \log(2\pi \cdot V_*) + \frac{(r_i - r_*)^2}{V_*} \right\}
\]
Estimating the Predictive Variance – A Significant Step for GDM?

Importance for Modelling Gas Distributions
5 Benefits of Predictive Variance

- Spatial Structure of Concentration Variance

1. Important information about gas distributions (fluctuations)
Estimating the Predictive Variance – A Significant Step for GDM?

Model Evaluation
5 Benefits of Predictive Variance

- **Ground Truth Evaluation**
  - capability to infer hidden parameters can be used
    - source location
    - independently measured mean concentration
  - no clear correspondence between source location and maximum of distribution mean
  - independent measurements at the same height difficult in robotic experiments

- **Prediction Capability**
  - good model = allows to infer concentration levels "explains observations and accurately predict new ones"
5 Benefits of Predictive Variance

- Prediction Quality
  - negative log predictive density (NLPD) = average neg. log likelihood (assuming Gaussian posterior)

\[
NLPD = \frac{1}{2|D|} \cdot \sum_{i=1}^{D} \left\{ \log(2\pi \cdot V_\ast) + \frac{(r_i - r_\ast)^2}{V_\ast} \right\}
\]

- an estimate of pred. uncertainty (variance) is necessary
5 Benefits of Predictive Variance

- Prediction Quality
  - negative log predictive density (NLPD)
    - = average neg. log likelihood (assuming Gaussian posterior)
    - \[ NLPD = \frac{1}{2|D|} \cdot \sum_{i=1}^{D} \left\{ \log(2\pi \cdot V_*) + \frac{(r_i - r_*)^2}{V_*} \right\} \]
  - an estimate of pred. uncertainty (variance) is necessary
  - enables parameter learning from data
5 Benefits of Predictive Variance

- **Prediction Quality**
  - negative log predictive density (NLPD)
    = average neg. log likelihood (assuming Gaussian posterior)

\[
NLPD = \frac{1}{2|D|} \cdot \sum_{i=1}^{D} \left\{ \log(2\pi \cdot V_*) + \frac{(r_i - r_*)^2}{V_*} \right\}
\]

- an estimate of pred. uncertainty (variance) is necessary
- enables parameter learning from data
- allows better ground truth evaluation
  - comparison of different approaches to GDM
5 Benefits of Predictive Variance

- Comparison: GPM with Kernel DM+V
  - learned via EM/CV (GPM), resp. CV (Kernel DM+V)

<table>
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<td>-0.90</td>
<td>-1.54</td>
<td>-1.44</td>
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<td>-0.98</td>
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5  Benefits of Predictive Variance

- Comparison: GPM with Kernel DM+
  - learned via EM/CV (GPM), resp. CV

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## 5 Benefits of Predictive Variance

- **Comparison:** GPM with Kernel DM+V, learned via EM/CV (GPM), resp. CV (Kernel DM+V)

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Kernel DM+V Extensions – Integration with SLAM
6 Integration with SLAM

- General SLAM Problem

\[ p(x^{1:t}, m^t \mid u^{1:t}, z^{1:t}) \]

- simultaneously estimate the map and the robot path given robot actions \( u \) and observations \( z \)
6 Integration with SLAM

- General SLAM Problem

\[ p(x^{1:t}, m^t \mid u^{1:t}, z^{1:t}) \]

simultaneously estimate the map and the robot path
given robot actions \( u \) and observations \( z \)

- Simultaneous Localisation and Gas and Occupancy Mapping (GDM/SLAM)

\[ m \leftarrow m = (m_{\text{gas}}, m_{\text{occ}}) \]

\[ z_t \leftarrow z_t = (z_{\text{gas},t}, z_{\text{occ},t}) \]
6 Integration with SLAM

- GDM/SLAM Problem
  - Rao-Blackwellized particle filter formulation

Kernel DM+V Extensions –
Learning Analytical Dispersal Models from Statistical GDMs
6 Statistical → Analytical Model Analysis

**Approach**

- interpret statistical GDM using a model from physics
- fit physical model of the average gas concentration to the gas distribution map

\[
\tilde{C}(x, y) = C_0 e^{-C_S r^2} e^{-C_A (r - [(x_S - x) \cos \theta + (y_S - y) \sin \theta])} + C_B
\]

**Assumptions**

- GDM represents the time-averaged gas concentration
- assumptions of the physical model hold
6 Statistical → Analytical Model Analysis

- **Fit of the Analytical Model**

\[
\tilde{C}(x, y) = C_0 e^{-C_S r^2} e^{-C_A (r - [(x_S - x) \cos \theta + (y_S - y) \sin \theta])} + C_B
\]
Fit of the Analytical Model

\[ \tilde{C}(x, y) = C_0 e^{-C_S r^2} e^{-C_A (r - [(x_S - x) \cos \theta + (y_S - y) \sin \theta])} + C_B \]
Kernel DM+V Extensions –
GDM with Multiple Gas Source
6 GDM with Multiple Gas Sources

Approach
6  GDM with Multiple Gas Sources

- Indoor/Outdoor Experiment
  - assuming a single source
6 GDM with Multiple Gas Sources

- Indoor/Outdoor Experiment
  - classification: transient, Wavelet decomposition, SVM

Ongoing and Future Work
7 Ongoing Work

- Diadem
  - larger environments, large sensor networks
7 Ongoing Work

- Diadem
  - larger environments, large sensor networks
  - sensor planning
    - where should the next measurements be carried out? (given the current model)
    - estimation of the required sensor density
  - estimation of the conditions under which statistical models are applicable
7 Ongoing and Future Work

- Time-dependent GDM
  - so far: assumption of a time-constant random process
  - regression approaches can be extended by time dimension
  - density estimation approaches with recency weights
    - use NLPD to learn appropriate recency weights
  - addition of a method to determine the appropriate time-window over which the distribution model is computed
  - lazy update mechanism depending on the NLPD ...
    - ... from cross-validation over the set of measurements (internal consistency)
    - ... over a set of predictions of new samples
Further Ongoing Work

- larger environments, large sensor networks
- optimal sensor planning
  - estimation of the required sensor density
- estimation of the conditions under which statistical models are applicable
- including wind information
Further Ongoing Work

- larger environments, large sensor networks
- optimal sensor planning
  - estimation of the required sensor density
- estimation of the conditions under which statistical models are applicable
  - including wind information
- 3-d gas distribution maps

Matteo Reggente: 3D Statistical Gas Distribution Mapping in an Uncontrolled Indoor Environment
Aula Magna, 17:30 o'clock
Further Ongoing Work

- larger environments, large sensor networks
- optimal sensor planning
  - estimation of the required sensor density
- estimation of the conditions under which statistical models are applicable
  - including wind information
- 3-d gas distribution maps
- 4-d gas distribution maps
  - model gas distribution at different time scales
Future Work

getting rid of the Gaussian posterior assumption

\[
m^{(ij)} = \frac{1}{B(\alpha_1, \ldots, \alpha_N)} \prod_{n=1}^{N} \left( \mu_n^{(ij)} \right)^{\alpha_n^{(ij)}} - 1
\]
7 Future Work

- Future Work
  - getting rid of the Gaussian posterior assumption
    \[
    m^{(ij)} = \frac{1}{B(\alpha_1, \ldots, \alpha_N)} \prod_{n=1}^{N} (\mu_n^{(ij)})^{\alpha_n^{(ij)}-1}
    \]
  - including gas discrimination into gas distribution modelling
    - classification posterior $\Rightarrow$ modelling algorithm
7 Future Work

- Integration of Models from Physics
  - measure spatial outline, heat distribution, wind, etc.
Summary
8 Summary

1. Introduction
8 Summary

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2. Gas Distribution in a Natural Environment
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Statistical Approaches to Gas Distribution Modelling with Mobile Robots –
The Kernel DM+V Algorithm and Beyond

Thanks for your attention and mental participation!